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Rotation and Acceleration as Parallel Transport in a Space-Time with Torsion

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We demonstrate that two simple noninertial motions, namely, uniformly rotating and uniformly accelerated motions, can be described as parallel transports, in a space-time manifold with torsion, of the moving object's reference frame along its world line. On the basis of that, it is conjectured that the electromagnetic field tensor is really only the temporal part of the contortion tensor. The only disturbing feature is that the decrease of the electromagnetic/gravitational coupling ratio with velocity (by the factor γ) does not appear in this approach.

1. INTRODUCTION

The two basic assumptions of Einstein's theory of gravitation (general relativity theory) are as follows (see, e.g., Born, 1965, p. 350).

1. All the properties of the gravitational field can be described by the non-Euclidean (or rather non-Minkowskian) metric of the four-dimensional space-time manifold (corresponding to the classical gravitational potential), and/or by the Levi-Civita (torsionless, or sometimes called "symmetric") affine connection derived from this metric (corresponding to the classical gravitational force).

2. The world lines of test particles in a gravitational field are geodesics (lines of extremal length) which, for the Levi-Civita connection, coincide with autoparallels (the straightest lines).

As it was shown first by Einstein, Infeld, Hoffmann, and later others, the second hypothesis is not independent, but rather follows from Einstein's equations for the gravitational field.

Immediately two questions arise:

1. From the physical side, one looks for a way to describe geometrically other, noninertial, i.e., rotating and accelerated motions.[1]

2. From the geometrical side, one tries to find the physical meaning

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for the torsional part of the Cartan's (sometimes referred to as "asymmetric") affine connection, which causes autoparallels to differ from the extremals; even in a flat (Minkowskian) space-time, autoparallels are "bent" (see Gogala, 1980a). One thus tries to find what prominent physical lines can be identified with the most prominent geometrical lines in a space-time endowed with torsion.

The aim of this paper is to demonstrate that the world lines of test particles exhibiting some simple, noninertial motions, like uniformly rotating and uniformly accelerated, can be described as autoparallels in a space-time with torsion. In other words, the "time direction" of an observer, associated with such a particle, remains, in a space-time with torsion, parallel to itself along the observer's world line. But not only that; the other three, space-directed vectors of the observer's tetrad are also moved parallel to themselves along the observer's world line, as prescribed by the torsion of the space-time.

We must stress that in this treatment the space-time is assumed to be flat (Minkowskian). Thus, in Cartesian coordinates, all the components of the affine connection are expressed in terms of the components of the torsion only, as the Christoffel symbols are identically zero. We will assume, however, the validity of the equivalence principle, which includes the property that the covariant derivative of the metric tensor is zero (see von der Heyde, 1975; Gogala, 1980a). The components of the affine connection in Cartesian coordinates will then be

$$\Gamma_{klm} = S_{mkl} \tag{1.1}$$

where the contortion tensor

$$S_{mkl} = \tfrac{1}{2}(T_{mkl} + T_{lkm} - T_{klm}) \tag{1.2}$$

is antisymmetric in the last two indices. The equations for parallel transport of vector $\mathbf{v}$, along a curve with tangent vector $\mathbf{u}$, will read in the same coordinates (cf. Gogala, 1980a):

$$\nabla_{\mathbf{u}}\mathbf{v} = u^m\left(v^k{}_{/m} + S_m{}^k{}_l v^l\right)\mathbf{e}_k = \left(\partial_{\mathbf{u}} v^k + u^m S_m{}^k{}_l v^l\right)\mathbf{e}_k = 0 \tag{1.3}$$

In our case, vector $\mathbf{u}$ will be the timelike vector, tangent to the observer's world line, and vector $\mathbf{v}$ will be one of the four vectors of his tetrad.

[1]It is sometimes claimed that accelerated motion is described by Einstein's general relativity theory. This is not true; the general relativity theory describes only inertial motions, albeit in curved space-time. Accelerated motion in a strict sense, that is, caused by an interaction other than gravitational, e.g., electromagnetic interaction, is noninertial.

2. UNIFORMLY ROTATING MOTION

For didactical reasons, the uniformly rotating motion although being more complicated, will be treated first. We find first the reference frame (tetrad) of the rotating observer by performing a Lorentz transformation on the inertial's observer's reference frame.[2]

As the rotating observer's reference frame is noninertial, this transformation can be only instantaneous. In rotating cylindrical coordinates, the instantaneous inertial reference frame is

$$\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial r}, \frac{\partial}{\partial \theta}, \frac{\partial}{\partial z}\right) = (\mathbf{e}_t, \mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z) \tag{2.1}$$

This frame keeps changing with time as

$$\mathbf{e}_r = \cos(\omega t)\mathbf{e}_x + \sin(\omega t)\mathbf{e}_y \tag{2.2a}$$

$$\mathbf{e}_\theta = r\left[-\sin(\omega t)\mathbf{e}_x + \cos(\omega t)\mathbf{e}_y\right] \tag{2.2b}$$

The motion in the three-dimensional space is in the direction $\mathbf{e}_\theta$, so only the vectors $\mathbf{e}_t$ and $\mathbf{e}_\theta$ will be affected by the Lorentz transformation. The vectors of the rotating observer's reference frame are thus

$$\mathbf{T} = \gamma \mathbf{e}_t + \gamma\omega \mathbf{e}_\theta = \gamma \mathbf{e}_t + \gamma\omega r\left[-\sin(\omega t)\mathbf{e}_x + \cos(\omega t)\mathbf{e}_y\right] \tag{2.3a}$$

$$\mathbf{R} = \mathbf{e}_r = \cos(\omega t)\mathbf{e}_x + \sin(\omega t)\mathbf{e}_y \tag{2.3b}$$

$$\mathbf{\Theta} = \gamma\omega r^2 \mathbf{e}_t + \gamma \mathbf{e}_\theta = \gamma\omega r^2 \mathbf{e}_t + \gamma r\left[-\sin(\omega t)\mathbf{e}_x + \cos(\omega t)\mathbf{e}_y\right] \tag{2.3c}$$

$$\mathbf{Z} = \mathbf{e}_z \tag{2.3d}$$

Here,

$$\gamma = (1 - \omega^2 r^2)^{-1/2} \tag{2.4}$$

[2]We must stress here the difference between the coordinate system and the reference frame (or frame of reference). A set of four variables x^i, which uniquely labels every event in the space-time continuum, is a coordinate system. A reference frame is defined at every point as an orthonormal tetrad of one timelike and three spacelike vectors. The former is interpreted as the 4-velocity vector of an observer at that point, and the latter as his local Cartesian coordinate axes (see Irvine, 1964; Pirani, 1957). The coordinate systems and reference frames are completely unrelated to each other, although a unique reference frame is naturally associated in each point with the given coordinate system, and although one can always find a coordinate system associated in a particular point with a given reference frame (making metric Minkowskian at that point). Lorentz transformations act only on the vectors of the reference frame, but not on the coordinate system, which is acted upon by general coordinate transformations. A particularly nice, detailed explanation of this subject was given in a series of papers by Rodichev (1965, 1967, 1968, 1972, 1976).

This result has also been found and discussed by Strauss (1974), and is identical to that of Irvine (1964), who used Frenet–Serret formulas instead of Lorentz transformations. The frame (2.3) is, of course, also orthonormal, as is the inertial frame of the static observer (2.1), but with the difference that it is anholonomic, as already discussed by Corum (1977); this is due to the transformation being instantaneous, that is, only local geometrically. The nonzero commutators are

$$[\mathbf{R},\mathbf{\Theta}] = -\gamma^2\omega^2 r\mathbf{\Theta} + 2\gamma^2\omega r\mathbf{T} \tag{2.5a}$$

$$[\mathbf{R},\mathbf{T}] = \gamma^2\omega^2 r\mathbf{T} \tag{2.5b}$$

The space-time remains flat, as no coordinate or basis transformations can make a flat manifold become curved. All the discussion in the literature about the surface of a rotating disk being curved owing to the length contraction of its circumference (or rather of all the concentric circles on it) makes no sense; the appearance of curvature arises from neglecting the fact that the space-time of a rotating observer is anholonomic, in other words, from forgetting the last term, which is nonzero owing to (2.5), in the expression for the curvature tensor

$$\begin{aligned} Q^x{}_{yuv} &= \hat{\Gamma}^x{}_{yv/u} - \hat{\Gamma}^x{}_{yu/v} \\ &\quad + \hat{\Gamma}^x{}_{mu}\hat{\Gamma}^m{}_{yv} - \hat{\Gamma}^x{}_{mv}\hat{\Gamma}^m{}_{yu} - \hat{\Gamma}^x{}_{ym}c^m{}_{uv} \end{aligned} \tag{2.6}$$

We will also need the derivatives of the components of all the vectors (2.3) of the rotating observer's frame with respect to the Galilean basis vectors:

$$\begin{aligned}
&T^t{}_{/x} = \gamma^2\omega T^y, \qquad T^x{}_{/t} = -\omega T^y \\
&T^t{}_{/y} = -\gamma^2\omega T^x, \qquad T^y{}_{/t} = \omega T^x \\
&T^x{}_{/x} = (-\omega/\gamma)\sin\omega t\cos\omega t, \qquad T^y{}_{/x} = (\omega/\gamma)\cos^2\omega t \\
&T^x{}_{/y} = (-\omega/\gamma)\sin^2\omega t, \qquad T^y{}_{/y} = (\omega/\gamma)\sin\omega t\cos\omega t \\
&R^x{}_{/t} = -\omega R^y, \qquad \Theta^t{}_{/x} = (1+\gamma^2)\gamma\omega r\cos\omega t \\
&R^y{}_{/t} = \omega R^x, \qquad \Theta^t{}_{/y} = (1+\gamma^2)\gamma\omega r\sin\omega t \\
&\Theta^x{}_{/t} = -\omega\Theta^y, \qquad \Theta^x{}_{/x} = (-1/\gamma)\sin\omega t\cos\omega t \\
&\Theta^y{}_{/t} = \omega\Theta^x, \qquad \Theta^y{}_{/y} = (1/\gamma)\sin\omega t\cos\omega t \\
&\Theta^x{}_{/y} = (-1/\gamma)\sin^2\omega t, \qquad \Theta^y{}_{/x} = (1/\gamma)\cos^2\omega t
\end{aligned} \tag{2.7}$$

All the other derivatives are identically zero. Here we had to take into account the fact that γ is not constant, but

$$\gamma_{/r} = \gamma^3\omega^2 r \tag{2.8}$$

We now insert the above values (2.7) into the formula for parallel transport (1.3), and find out what requirements this formula imposes on the components of the contortion tensor. Obviously, vector **Z** does not change in any way, and also none of the other three vectors depend in any way on the coordinate z. So, all the components of the contortion tensor with at least one index z are zero.

For the vector $\mathbf{v}=\mathbf{T}$, we get, for $\mathbf{e}_l=\mathbf{e}_t$,

$$T^tT^x(S_{ttx}) + T^tT^y(S_{tty}) + T^xT^x(S_{xtx}) + T^yT^y(S_{yty}) + T^xT^y(S_{xty} + S_{ytx}) = 0 \tag{2.9a}$$

for $\mathbf{e}_l=\mathbf{e}_x$,

$$T^tT^t(S_{txt}) + T^tT^x(S_{xxt}) + T^xT^y(S_{xxy}) + T^yT^y(S_{yxy}) + T^tT^y(-\omega + S_{txy} + S_{yxt}) = 0 \tag{2.9b}$$

for $\mathbf{e}_l=\mathbf{e}_y$,

$$T^tT^t(S_{tyt}) + T^tT^y(S_{yyt}) + T^xT^y(S_{yyx}) + T^xT^x(S_{xyx}) + T^tT^x(\omega + S_{tyx} + S_{xyt}) = 0 \tag{2.9c}$$

As these requirements must be fulfilled for any time t, and as the terms in front of the brackets in (2.9) each depend on t in a different way, the expressions in brackets must be zero. So

$$\begin{gathered} S_{ttx} = S_{tty} = S_{xtx} = S_{yty} = S_{xxy} = S_{yyx} = 0 \\ -\omega + S_{txy} + S_{yxt} = 0, \qquad \omega - S_{txy} + S_{xyt} = 0 \\ S_{xyt} + S_{yxt} = 0 \end{gathered} \tag{2.10}$$

For the vector $\mathbf{v}=\mathbf{R}$, we find, for $\mathbf{e}_l=\mathbf{e}_t$,

$$T^tR^x(S_{ttx}) + T^tR^y(S_{tty}) + T^xR^y(S_{xty}) + T^yR^x(S_{ytx}) + T^xR^x(S_{xtx}) + T^yR^yS_{yty} = 0 \tag{2.11a}$$

for $\mathbf{e}_l = \mathbf{e}_x$,

$$T^t R^y(-\omega + S_{t\,xy}) + T^x R^y(S_{x\,xy}) + T^y R^y(S_{y\,xy}) = 0 \tag{2.11b}$$

for $\mathbf{e}_l = \mathbf{e}_y$,

$$T^t R^x(\omega + S_{tyx}) + T^x R^x(S_{xyx}) + T^y R^y(S_{yyx}) = 0 \tag{2.11c}$$

Reasoning in the same way as above, we find

$$S_{t\,tx} = S_{t\,ty} = S_{x\,ty} = S_{y\,tx} = S_{x\,xy} = S_{y\,xy} = 0$$

$$-\omega + S_{t\,xy} = 0, \qquad S_{x\,tx} - S_{y\,ty} = 0 \qquad (\text{because } T^x R^x = -T^y R^y) \tag{2.12}$$

For the vector $\mathbf{v} = \boldsymbol{\Theta}$, we find, for $\mathbf{e}_l = \mathbf{e}_t$,

$$T^t \Theta^x(S_{t\,tx}) + T^t \Theta^y(S_{t\,ty}) + T^x \Theta^x(S_{x\,tx}) + T^y \Theta^y(S_{y\,ty}) + T^x \Theta^y(S_{x\,ty}) + T^y \Theta^x(S_{y\,tx}) = 0 \tag{2.13a}$$

for $\mathbf{e}_l = \mathbf{e}_x$,

$$T^t \Theta^y(-\omega + S_{t\,xy}) + T^y \Theta^t(S_{y\,xt}) + T^t \Theta^t(S_{t\,xt}) + T^y \Theta^y(S_{y\,xy}) + T^x \Theta^y(S_{x\,xy}) + T^x \Theta^t(S_{x\,xt}) = 0 \tag{2.13b}$$

for $\mathbf{e}_l = \mathbf{e}_y$,

$$T^t \Theta^x(\omega + S_{tyx}) + T^x \Theta^t(S_{xyt}) + T^t \Theta^t(S_{tyt}) + T^x \Theta^x(S_{xyx}) + T^y \Theta^y(S_{yyx}) + T^y \Theta^t(S_{yyt}) = 0 \tag{2.13c}$$

and again

$$S_{t\,tx} = S_{t\,ty} = S_{x\,tx} = S_{y\,ty} = S_{x\,xy} = S_{y\,xy} = 0$$

$$S_{x\,ty} - S_{y\,tx} = 0 \qquad (\text{because } T^x \Theta^y = -T^y \Theta^x)$$

$$-\omega + S_{t\,xy} + \omega^2 r^2 S_{y\,xt} = 0 \qquad (\text{because } T^y \Theta^t = \omega^2 r^2 T^t \Theta^y)$$

$$\omega - S_{t\,xy} + \omega^2 r^2 S_{xyt} = 0 \qquad (\text{because } T^x \Theta^t = \omega^2 r^2 T^t \Theta^x) \tag{2.14}$$

Wrapping it all together, we conclude that all the components of the contortion tensor are zero, except

$$S_{t\,xy} = \omega \tag{2.15}$$

The only known field, which is homogeneous and which can make a particle go around in a circle, is the magnetic field; so we can identify, up to the factor e_0/m_0,

$$S_{txy} = -B_z = -F_{xy} \tag{2.16}$$

The minus sign is caused by the fact that the magnetic field in the negative direction z causes the rotation to be counterclockwise.

We know from experience that in reality ω decreases with higher velocities and constant magnetic field. It appears that magnetic field is reduced by a factor γ, if the coupling is assumed to be constant. As the field obviously cannot be reduced, just because some particle started to move faster, it follows that either the coupling between the charge and the magnetic field decreases with velocity, or the coupling between the mass and the surrounding metric field of space-time increases with velocity.

3. UNIFORMLY ACCELERATED MOTION

We consider now a uniformly accelerated observer, i.e., an observer performing hyperbolic motion. His world line is described (see Misner, Thorne, and Wheeler, 1973, equation 6.5) by

$$t = (1/a)\sinh a\tau, \qquad z = (1/a)\cosh a\tau \tag{3.1}$$

Here a is his acceleration, and τ is his proper time,

$$\tau = \frac{1}{2a}\ln\frac{z+t}{z-t} \tag{3.2}$$

and

$$\gamma = \cosh a\tau, \qquad \beta\gamma = \sinh a\tau \tag{3.3}$$

The reference frame of the accelerated observer is expressed in terms of the reference frame of an inertial observer,

$$\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = (\mathbf{e}_t, \mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z) \tag{3.4}$$

by (see Misner et al., 1973, equation 6.6)

$$\mathbf{T} = \cosh(a\tau)\mathbf{e}_t + \sinh(a\tau)\mathbf{e}_z = az\mathbf{e}_t + at\mathbf{e}_z \tag{3.5a}$$

$$\mathbf{X} = \mathbf{e}_x, \qquad \mathbf{Y} = \mathbf{e}_y \tag{3.5b}$$

$$\mathbf{Z} = \sinh(a\tau)\mathbf{e}_t + \cosh(a\tau)\mathbf{e}_z = at\mathbf{e}_t + az\mathbf{e}_z \tag{3.5c}$$

The reference frame of the accelerated observer is, of course, still metric, and, unlike the previous case, also holonomic:

$$[\mathbf{T}, \mathbf{Z}] = 0 \tag{3.6}$$

The derivatives of the components of the vectors of the accelerated observer's reference frame are

$$T^t{}_{/z} = T^z{}_{/t} = Z^t{}_{/t} = Z^z{}_{/z} = a \tag{3.7}$$

We again insert all these values into equation (1.3), and find the conditions on the components of the contortion tensor. Obviously, now all the components with at least one index x or y will be zero.

For $\mathbf{v} = \mathbf{T}$, we get, for $\mathbf{e}_l = \mathbf{e}_t$,

$$-T^t T^z S_{ttz} + T^z(a + T^z S_{zzt}) = 0 \tag{3.8a}$$

for $\mathbf{e}_l = \mathbf{e}_z$,

$$T^t(a + T^t S_{tzt}) + T^z T^t S_{zzt} = 0 \tag{3.8b}$$

and for $\mathbf{v} = \mathbf{Z}$, we get, for $\mathbf{e}_l = \mathbf{e}_t$,

$$T^t(a - Z^z S_{ttz}) - T^z Z^z S_{ztz} = 0 \tag{3.9a}$$

for $\mathbf{e}_l = \mathbf{e}_z$,

$$T^t Z^t S_{tzt} + T^z(a + Z^t S_{zzt}) = 0 \tag{3.9b}$$

Only one of these equations is really independent, because we are finally left with the following:

from (3.8a) and (3.9b),

$$a^2 t(1 - zS_{ttz} - tS_{ztz}) = 0 \tag{3.10a}$$

and from (3.8b) and (3.9a),

$$a^2 z(1 - zS_{ttz} - tS_{ztz}) = 0 \tag{3.10b}$$

The situation is now less straightforward than before. For $t = 0$, it is easy:

$$S_{ttz}(t=0) = \frac{1}{z(t=0)} = a \tag{3.11}$$

and $S_{z\,tz}$ is undetermined. Obviously, the torsion of the space-time cannot depend on the motion of a particle in it. We must therefore assume that $S_{z\,tz}$ is zero at all times, and then

$$S_{t\,tz} = \frac{1}{z} = \frac{a}{\cosh a\tau} = \frac{a}{\gamma} \tag{3.12}$$

Thus, for an accelerated observer, the contortion of the manifold, in which the this observer moves his reference frame parallel to itself along his world line, again appears reduced by the factor γ. As the electric field is the only known field which can accelerate a particle, and which also appears reduced by the factor γ, we can identify

$$S_{t\,tz} = E_z = -F_{tz} \tag{3.13}$$

4. DISCUSSION

It must be remarked that both the above results are not entirely new. Rogozhin (1971) obtained the same results, using the language of group deformations; he did not use the contortion tensor, but expressed his results in terms of the torsion tensor components. As this author is not familiar with the language of group deformations, he was not able to determine whether Rogozhin had found also that the space-directed vectors of the noninertial observer's reference frame move parallel to themselves along his world line, or only that the world lines are autoparallels in a space-time with torsion.

We found that the contortion of space-time can describe simple noninertial motions as parallel transport of the comoving observer's reference frame along his world line. The contortion tensor takes care of correctly rotating not only the timelike velocity vector but also all the other three spacelike vectors of the moving body's reference frame.

We observe that there is an analogy between the noninertial motion, caused by the electromagnetic field, and the motion in the gravitational field. In both cases we have rotation of the orthonormal axes of the moving observer's reference frame when they are transported parallel along his world line. The geometrical cause of these rotations is, however, different in both cases. In the former case, the cause is contortion of space-time, and in the latter case the cause is Riemannian structure of space-time which, in the local anholonomic basis of the moving observer, finds its expression in the Ricci rotation coefficients (for a more detailed discussion about these two rotations see Hehl et al., 1976, p. 398, and

Gogala, 1980a). It is apparently this latter rotation, and not rotation due to real acceleration, which was treated by Misner, Thorne, and Wheeler (1973, Section 13.6).

We also showed that we can identify some components of the contortion tensor with the components of the electromagnetic field. The only disturbing feature is the factor γ, which makes the field appear reduced at higher velocity. A possible explanation for this discrepancy will be given in another paper (Gogala, 1980b), which will also deal with extending the above results to obtain a unified field theory, based on the contortion tensor of the space-time manifold.

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